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FREE CONVECTION AT SMALL RAYLEIGH NUMBER IN POROUS CAVITIES OF RECTANGULAR, ELLIPTICAL, TRIANGULAR AND OTHER CROSS-SECTIONS

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Abstract—Exact solutions are developed for small Rayleigh number free convection in 2-dim. porous cavities of various shapes due to a uniform temperature gradient normal to the gravitational field. The theorem is established that such flows are independent of cavity orientation. The appropriate (rotation-invariant) characteristic length is thus $A^{1/2}$ (A, cross-sectional area). Exact solutions are presented in detail for rectangular and elliptical cavities with λ (ratio of long to short axis) arbitrary, and for equilateral triangular cavities. The critical Rayleigh number for applicability of the analysis is about 1 for cavities with long axis horizontal, and about λ^2 with long axis vertical. Simple exact solutions exist also for cavities of other shapes.

NOMENCLATURE

A, cross-sectional area of porous cavity;
a, for rectangular cavity, length of longer side; for elliptical cavity, length of major axis;
b, for rectangular cavity, length of shorter side; for elliptical cavity, length of minor axis;
g, gravitational acceleration;
H, total convective heat transport within the cavity across the horizontal containing the cavity centre;
K, total conductive heat transport within the cavity across the vertical containing the cavity centre;
k, permeability of the porous medium;
L, characteristic macroscopic length of the system;
l, for equilateral triangle, length of side;
R, Rayleigh number;
R_1, R_2, maximum Rayleigh number for applicability of present analysis with x_* direction respectively horizontal and vertical;
T, dimensionless temperature;
ΔT_*, characteristic macroscopic horizontal temperature difference of the system;
U, W, dimensionless flow velocity in the X_*, Z_* directions;
X, Z, dimensionless rectangular Cartesian coordinates in the cross-sectional plane of the cavity, with Z vertical, positive upwards;
x, z, dimensionless rectangular Cartesian coordinates in the cross-sectional plane of the cavity, with coordinate directions fixed by cavity geometry, not the gravitational field.

Greek symbols

α, coefficient of thermal volume expansion;
β, ratio of convective to conductive heat transfer;
θ, minus constant horizontal temperature gradient;
κ, thermal diffusivity of the porous medium;
λ, aspect ratio, a/b;
ν, kinematic viscosity of the fluid in the porous medium;
Φ, dimensionless stream function;
Φ_{max}, maximum value of Φ.

Subscripts

1, value with x_* direction horizontal;
2, value with x_* direction vertical;
*, dimensional rather than dimensionless variable.

1. INTRODUCTION

THERE is an extensive literature on free convection in porous media, i.e. on fluid flows in porous media in a gravitational field which are driven by gradients of fluid density caused by gradients of temperature or solute concentration. Numerous applications occur in industrial and geophysical contexts. In the present work we refer specifically to temperature-induced flows, but the modifications to solute-induced flows will be obvious and straightforward.

Many studies, including most of the earlier work, have dealt with systems heated from below [1–8]. Recently much attention has been given to investigation of free convection in porous media induced by a temperature gradient normal to the gravitational field [9–15]. Almost all these studies relate to the 2-dim. problem of steady free convection at large Rayleigh number in a rectangular cavity with two opposite vertical boundaries held at fixed but different temperatures. This has become the paradigmatic problem, and I am unaware of comparable 2-dim. studies for other cavity shapes.

There appear to have been no analogous studies of free convection in porous cavities at small Rayleigh number, even for the rectangular cavity. The present work develops exact solutions for small Rayleigh number free convection due to a temperature gradient normal to the gravitational field in 2-dim. porous cavities of various shapes. Despite their simplicity, these convection solutions do not seem to have been recognized previously.

Although virtually all work has been done on large Rayleigh number flows, small Rayleigh number convection is also of practical interest and concern. Section 7.2 discusses the application to soils. The mathematics of small Rayleigh number flows might be considered so elementary as to be without interest. Yet it is remarkable that relatively complicated flows can be analyzed so simply; and it must be added that small Rayleigh number solutions represent natural points of departure for solutions obtained by regular expansion in the Rayleigh number. For example, the exact solution presented in equation (3.3) below is relevant to a recent essay at regular expansion of the rectangular cavity problem [14].

2. FLOW EQUATION FOR FREE CONVECTION AT SMALL RAYLEIGH NUMBER

The equations governing 2-dim. steady convection within a porous medium induced by a temperature differential may be written [4]

$$\nabla_*^2 \Phi_* = \frac{\alpha k g}{\nu}\frac{\partial T_*}{\partial X_*}, \tag{2.1}$$

$$\frac{\partial(\Phi_*, T_*)}{\partial(X_*, Z_*)} = \kappa \nabla_*^2 T_*, \tag{2.2}$$

where the stream function Φ_* is such that

$$U_* = -\frac{\partial \Phi_*}{\partial Z_*}, \quad W_* = \frac{\partial \Phi_*}{\partial X_*}. \tag{2.3}$$

We introduce the Rayleigh number

$$R = \frac{\alpha k g \Delta T_* L}{\kappa \nu}. \tag{2.4}$$

Then equations (2.1)–(2.3) may be reduced to the dimensionless forms

$$\nabla^2 \Phi = R\frac{\partial T}{\partial X}, \tag{2.5}$$

$$\frac{\partial(\Phi, T)}{\partial(X, Z)} = \nabla^2 T, \tag{2.6}$$

$$U = -\frac{\partial \Phi}{\partial Z}, \quad W = \frac{\partial \Phi}{\partial X}. \tag{2.7}$$

Note that

$$\frac{X}{X_*} = \frac{Z}{Z_*} = \frac{1}{L}; \quad \frac{T}{T_*} = \frac{1}{\Delta T_*};$$

$$\frac{\Phi}{\Phi_*} = \frac{1}{\kappa}; \quad \frac{U}{U_*} = \frac{W}{W_*} = \frac{L}{\kappa}. \tag{2.8}$$

When R is so small that heat transfer by conduction dominates that due to convection, the RHS of equation (2.6) may be set equal to zero. Then if the horizontal temperature gradient in the system, $\partial T/\partial X_*$, is everywhere constant and equal to $-\theta$, we may rewrite equation (2.5) as

$$\nabla^2 \Phi + R = 0 \tag{2.9}$$

where we have replaced $\Delta T_* L$ on the RHS of equation (2.4) by θL^2.

Note that equation (2.9) is invariant under rotation in the (X, Y) plane. Accordingly, we have the following theorem:

Theorem. Steady 2-dim. free convection at low Rayleigh number due to a uniform horizontal temperature gradient within a porous cavity of arbitrary configuration is independent of the orientation of the cavity cross-section with respect to the gravitational field.

It follows that L^2 depends only on the shape and size of the cross-section and is independent of its orientation. Consequently we take $L^2 = A$, the cross-sectional area. We therefore restate equation (2.4) in the form

$$R = \frac{\alpha k g \theta A}{\kappa \nu}. \tag{2.10}$$

3. SOLUTION FOR RECTANGULAR CAVITIES

The rectangular cavity has sides of length a and b ($a \geqslant b$), and we choose coordinates (x_*, z_*) such that $x_* = \pm\frac{1}{2}a$ contain the short sides of the rectangle and $z_* = \pm\frac{1}{2}b$ contain the long sides. Note that, because of the rotational invariance of the problem, we work here, and in succeeding sections, with space coordinates (x_*, z_*), fixed by the cavity geometry, rather than with coordinates (X_*, Z_*), fixed by the direction of the gravitational field. We shall use also the dimensionless coordinates (x, z) defined by

$$\frac{x}{x_*} = \frac{z}{z_*} = \frac{1}{L}. \tag{3.1}$$

We then have equation (2.9) subject to the conditions

$$\begin{aligned} \Phi &= 0, \quad |x| \leqslant \tfrac{1}{2}\lambda^{1/2}, \quad z = \pm\tfrac{1}{2}\lambda^{-1/2}; \\ \Phi &= 0, \quad x = \pm\tfrac{1}{2}\lambda^{1/2}, \ |z| < \tfrac{1}{2}\lambda^{-1/2}. \end{aligned} \tag{3.2}$$

Here $\lambda = a/b$ is the aspect ratio.

The solution of equation (2.9) subject to the conditions (3.2) is [16]

$$\Phi = \frac{R}{8\lambda}\left[1 - 4\lambda z^2 - \frac{32}{\pi^3}\sum_{n=0}^{\infty} \times \frac{(-1)^n \cosh[(2n+1)\pi\lambda^{1/2}x]\cos[(2n+1)\pi\lambda^{1/2}z]}{(2n+1)^3\cosh[(n+\frac{1}{2})\pi\lambda]}\right]. \tag{3.3}$$

Figure 1 depicts the stream function in the dimensionless form $R^{-1}\Phi(x, z)$ for $\lambda = 1$, 2 and 4.

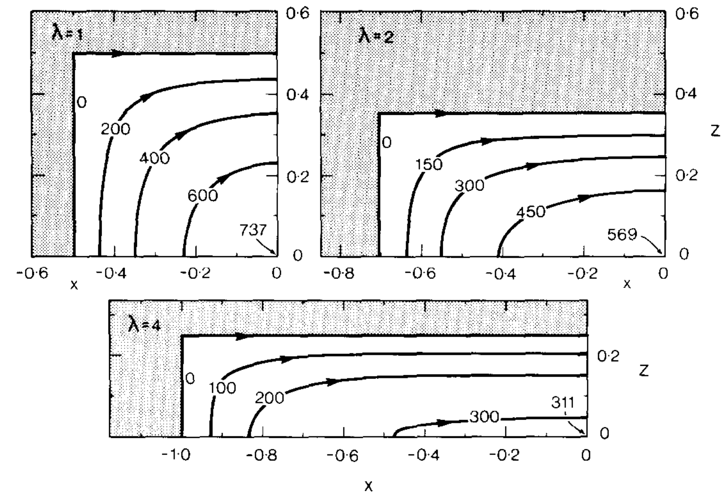

FIG. 1. Free convection at small Rayleigh number in rectangular cavities. Dimensionless plot of the stream function for aspect ratio $\lambda = 1$, 2, and 4. Numerals on the curves are values of $10^4\Phi/R$.

It follows from equation (3.3) that the maximum value of Φ, $\Phi_{\max} = \Phi(0, 0)$, is given by

$$\Phi_{\max} = \frac{R}{8\lambda}\left[1 - \frac{32}{\pi^3}\sum_{n=0}^{\infty}(-1)^n\frac{\operatorname{sech}[(n+\frac{1}{2})\pi\lambda]}{(2n+1)^3}\right]. \tag{3.4}$$

The series converges rapidly, so that replacing it by its leading term gives an excellent approximation. In fact, for $\lambda \geqslant 4.9$ the simple relation

$$\Phi_{\max} = \frac{R}{8\lambda} \tag{3.5}$$

is accurate to 1 in 1000. Figure 2 shows the dependence of $R^{-1}\,\Phi_{\max}$ on λ.

It is of interest to compare the exact solution for $\Phi_{\max}$, equation (3.4), with values secured for $\lambda = 1, 2, 3$ by a Galerkin method employing a truncated $(M \times N)$ spectral representation [14]. The exact values [requiring 2–5 terms of the summation on the right of

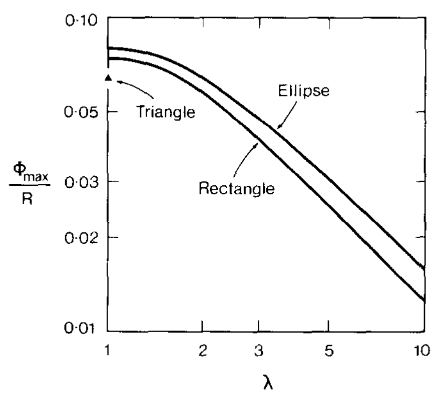

FIG. 2. Free convection at small Rayleigh number in rectangular, elliptical, and equilateral triangular cavities. Dependence of the maximum value of the dimensionless stream function on aspect ratio, plotted in the form $\Phi_{\max}/R$ against λ.

equation (3.4)] confirm the results obtained from the elaborate numerical study.

We now examine the relative magnitudes of convective and conductive heat transfer in the system. We take as the measure of convective transfer the quantity H, the total convective heat transport within the cavity across the horizontal containing the cavity centre (0, 0).

We denote by H_1 the value of H when the x_* direction is horizontal. It follows from equation (3.3) that

$$H_1 = \frac{\kappa\theta A^{1/2}R}{8\lambda^{1/2}}\left[1 - \frac{64}{\pi^4\lambda} \times \sum_{n=0}^{\infty}(-1)^n\frac{\tanh[(n+\frac{1}{2})\pi\lambda]}{(2n+1)^4}\right]. \tag{3.6}$$

Similarly, we denote by H_2 the value of H when the x_* direction is vertical. It follows from equation (3.3) that

$$H_2 = \frac{\kappa\theta A^{1/2}R}{12\lambda^{3/2}}\left[1 - \frac{96}{\pi^4}\sum_{n=0}^{\infty}\frac{\operatorname{sech}[(n+\frac{1}{2})\pi\lambda]}{(2n+1)^4}\right]. \tag{3.7}$$

We take as the measure of conductive heat transfer the quantity K, the total conductive heat transfer per unit axial length within the cavity across the vertical containing the cavity centre (0, 0). Denoting by K_1 the value of K with the x_* direction horizontal, we have

$$K_1 = \kappa\theta A^{1/2}\lambda^{-1/2}. \tag{3.8}$$

Similarly, the value of K with the x_* direction vertical,

$$K_2 = \kappa\theta A^{1/2}\lambda^{1/2}. \tag{3.9}$$

We introduce the ratio $\beta = H/K$ as the measure of the relative importance of convective and conductive heat transfer in the system; and, in particular, we write

Table 1. Comparison of numerical results [14] and present exact solution

	Values of $100\Phi_{max}$			
	Galerkin method using truncated spectral representation [14]			Exact value from equation (3.4)
λ	M	N	Value	
1	20	20	7.36556	7.367135
2	30	20	5.6921	5.693592
3	36	26	4.0905	4.089413

$\beta_1 = H_1/K_1$ and $\beta_2 = H_2/K_2$. We thus find

$$\beta_1 = \frac{R}{8}\left[1 - \frac{64}{\pi^4\lambda}\sum_{n=0}^{\infty}(-1)^n\frac{\tanh[(n+\frac{1}{2})\pi\lambda]}{(2n+1)^4}\right], \tag{3.10}$$

$$\beta_2 = \frac{R}{12\lambda^2}\left[1 - \frac{96}{\pi^4}\sum_{n=0}^{\infty}\frac{\operatorname{sech}[(n+\frac{1}{2})\pi\lambda]}{(2n+1)^4}\right]. \tag{3.11}$$

The criterion for applicability of the present analysis is that convective heat transfer should be small compared with conductive transfer, e.g. that $\beta < 0.1$. We thus obtain the following criteria on the Rayleigh number R:

For rectangular cavities with long axis horizontal

$$R < R_1 = \frac{4}{5}\left[1 - \frac{64}{\pi^4\lambda} \times \sum_{n=0}^{\infty}(-1)^n\frac{\tanh[(n+\frac{1}{2})\pi\lambda]}{(2n+1)^4}\right]^{-1}. \tag{3.12}$$

For rectangular cavities with long axis vertical

$$R < R_2 = \frac{6\lambda^2}{5}\left[1 - \frac{96}{\pi^4}\sum_{n=0}^{\infty}\frac{\operatorname{sech}[(n+\frac{1}{2})\pi\lambda]}{(2n+1)^4}\right]^{-1}. \tag{3.13}$$

The series in equations (3.12) and (3.13) converge rapidly, so that replacing them by their leading terms gives good approximations.

When λ is large enough, the simple expressions

$$R_1 = \frac{4}{5}\frac{\lambda}{\lambda - 0.65} \tag{3.14}$$

and

$$R_2 = 6\lambda^2/5 \tag{3.15}$$

are useful. Note that

$$\frac{64}{\pi^4}\sum_{n=0}^{\infty}(-1)^n(2n+1)^{-4} \approx 0.65.$$

For the square cavity with $\lambda = 1$, $\Phi_{max} = 0.0737\,R$, $\beta_1 = \beta_2 = R/19.8$, and $R_1 = R_2 = 1.98$.

The dependence of R_1 and R_2 on λ for rectangular cavities is shown in Fig. 3.

4. SOLUTION FOR ELLIPTICAL CAVITIES

The elliptical cavity has axes of length a and b ($a \geqslant b$) and we choose coordinates (x_*, z_*) such that $x_* = 0$ and $z_* = 0$ contain the minor and major axes respectively. In this case $L = \frac{1}{2}(\pi ab)^{1/2}$. We then have equation (2.9) subject to the condition that

$$\Phi = 0 \text{ on the ellipse } \pi\lambda^{-1}x^2 + \pi\lambda z^2 = 1. \tag{4.1}$$

The solution of equation (2.9) satisfying condition (4.1) is

$$\Phi = \frac{R}{2\pi}\frac{\lambda}{\lambda^2+1}(1 - \pi\lambda^{-1}x^2 - \pi\lambda z^2). \tag{4.2}$$

Figure 4 depicts the stream function in the dimensionless form $R^{-1}\Phi(x, z)$ for $\lambda = 1$, 2 and 4.

It follows from equation (4.2) that

$$\Phi_{max} = \frac{R}{2\pi}\cdot\frac{\lambda}{\lambda^2+1}. \tag{4.3}$$

The dependence of $R^{-1}\Phi_{max}$ on λ for elliptical cavities is shown on Fig. 2.

In this case

$$H_1 = \kappa\theta A^{1/2}\left(\frac{2R}{3\pi^{3/2}}\right)\left(\frac{\lambda^{3/2}}{\lambda^2+1}\right), \tag{4.4}$$

$$H_2 = \kappa\theta^{1/2}A^{1/2}\left(\frac{2R}{3\pi^{3/2}}\right)\left(\frac{\lambda^{1/2}}{\lambda^2+1}\right), \tag{4.5}$$

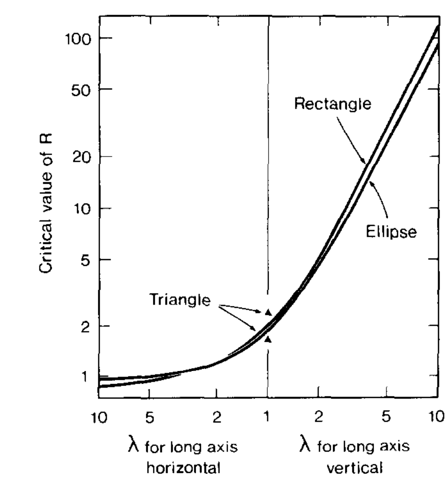

FIG. 3. Free convection at small Rayleigh number in rectangular, elliptical, and equilateral triangular cavities. Dependence on the aspect ratio λ of the critical value of R for applicability of the small Rayleigh number analysis.

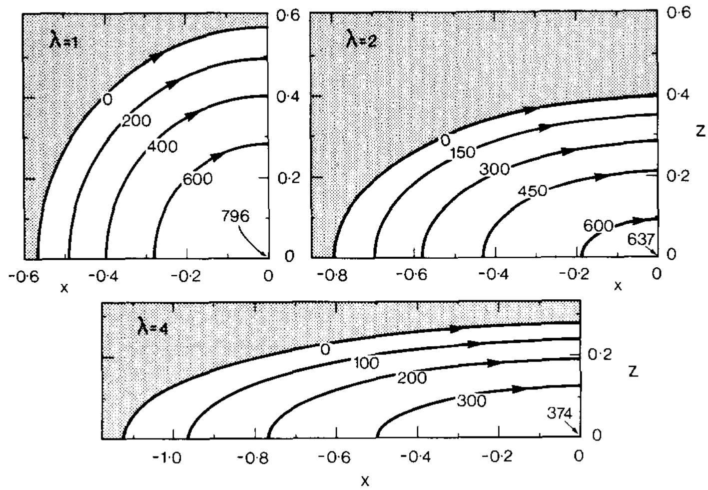

FIG. 4. Free convection at small Rayleigh number in elliptical cavities. Dimensionless plot of the stream function for aspect ratio $\lambda = 1, 2$, and 4. Numerals on the curves are values of $10^4\Phi/R$.

$$K_1 = \frac{2}{\pi^{1/2}}\kappa\theta A^{1/2}\lambda^{-1/2}, \tag{4.6}$$

$$K_2 = \frac{2}{\pi^{1/2}}\kappa\theta A^{1/2}\lambda^{1/2}. \tag{4.7}$$

We therefore have that

$$\beta_1 = \frac{R}{3\pi}\cdot\frac{\lambda^2}{\lambda^2+1} \tag{4.8}$$

and

$$\beta_2 = \frac{R}{3\pi}\cdot\frac{1}{\lambda^2+1} \tag{4.9}$$

and the criteria on R for applicability of the present analysis follow. For elliptical cavities with long axis horizontal

$$R < R_1 = \frac{3\pi}{10}(1+\lambda^{-2}). \tag{4.10}$$

For elliptical cavities with long axis vertical

$$R < R_2 = \frac{3\pi}{10}(\lambda^2+1). \tag{4.11}$$

For the circular cavity with $\lambda = 1$, $\Phi_{max} = 0.0796R$, $\beta_1 = \beta_2 = R/18.8$, and $R_1 = R_2 = 1.88$. The dependence of R_1 and R_2 on λ for elliptical cavities is shown in Fig. 3.

5. SOLUTION FOR TRIANGULAR CAVITIES

We consider the cavity with cross-section consisting of an equilateral triangle of side l. We choose coordinates (x_*, z_*) such that the centroid of the triangle is $(0,0)$ and one vertex is $(0, 3^{-1/2}l)$. The x_* axis is thus the perpendicular bisector of one side of the triangle. We have $A = \frac{1}{4}3^{1/2}l^2$, so that $L = \frac{1}{2}3^{1/4}l$. Equation (2.9) is then subject to the condition that

$$\Phi = 0 \text{ on the triangle } (z+3^{-3/4})(z-3^{1/2}x-2.3^{-3/4}) \times (z+3^{1/2}x-2.3^{-3/4}) = 0. \tag{5.1}$$

The required solution is

$$\Phi = \frac{R}{4.3^{1/4}}(z+3^{-3/4})(z-3^{1/2}x-2.3^{-3/4}) \times (z+3^{1/2}x-2.3^{-3/4}). \tag{5.2}$$

Figure 5 depicts the stream function in the dimensionless form $R^{-1}\Phi(x, z)$. We note that

$$\Phi_{max} = 3^{-5/2}R = 0.0642\,R. \tag{5.3}$$

This value is plotted on Fig. 2 for comparison with the result for rectangular and elliptical cavities.

In this case

$$H_1 = \kappa\theta A^{1/2}\cdot\frac{8R}{81.3^{3/4}}, \tag{5.4}$$

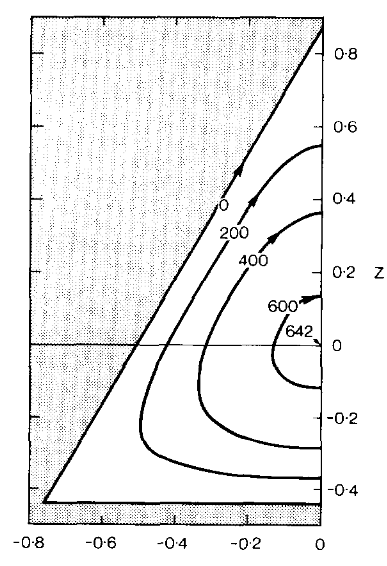

FIG. 5. Free convection at small Rayleigh number in equilateral triangular cavities. Dimensionless plot of the stream function. Numerals on the curves are values of $10^4\Phi/R$.

Table 2. Values of α, ν, κ for air- and water-filled soil at 293 K

Fluid	α (K^{-1})	ν ($m^2 s^{-1}$)	κ ($m^2 s^{-1}$)
Air	3.4×10^{-3}	1.53×10^{-5}	2×10^{-7}
Water	2.05×10^{-4}	1.01×10^{-6}	8×10^{-7}

$$H_2 = \kappa\theta A^{1/2} \cdot \frac{35R}{432.3^{1/4}}, \tag{5.5}$$

$$K_1 = 3^{1/4}\kappa\theta A^{1/2}, \tag{5.6}$$

$$K_2 = 4.3^{-5/4}\kappa\theta A^{1/2}. \tag{5.7}$$

Accordingly,

$$\beta_1 = \frac{8R}{243} = \frac{R}{30.4}, \tag{5.8}$$

$$\beta_2 = \frac{35R}{576} = \frac{R}{16.5}. \tag{5.9}$$

We thus have the following criteria for applicability of the present analysis:
For equilateral triangular cavities with one side horizontal

$$R < R_1 = 3.04. \tag{5.10}$$

For equilateral triangular cavities with one side vertical

$$R < R_2 = 1.65. \tag{5.11}$$

These criteria are shown on Fig. 3.

6. CAVITIES OF OTHER SHAPES

It will be evident from Sections 3–5 that exact solutions are readily available for cavities with various singly and doubly connected cross-sections bounded by streamlines in the preceding solutions. These cavity configurations are, in general, somewhat artificial, and we shall not elaborate on them here. For the cavity of *annular* cross-section we note in passing the simple result

$$\Phi_{max} = \frac{R}{4\pi}, \tag{6.1}$$

with the total convective flow independent of the ratio of the internal and external radii.

Simple exact solutions of equation (2.9) exist also for porous bodies of various infinite cross-sections. In general, these solutions are of restricted interest: physical realization of the boundary conditions at infinity is usually problematical; and the Rayleigh number tends to become arbitrarily large in the far flow field.

7. CONCLUDING REMARKS

7.1. *Comparison of results for cavities of various shapes*

Figures 1, 4, and 5 indicate that the general character of the convective flows for cavities of the various shapes and with various aspect ratios are essentially similar. We emphasize that $A^{1/2}$ proves a very efficient characteristic length, leading to dimensionless results for integral properties of the convective motion which vary relatively little with cavity shape. The variation of Φ_{max} with λ for rectangular and elliptical cavities is essentially similar. The ellipse constitutes a more efficient flow field than the rectangle, and the rectangle is more efficient than the triangle; and these differences are reflected in the result (Fig. 2) that Φ_{max} is greater for the ellipse than for the rectangle, and it is greater for both than for the triangle ($\lambda = 1$). As shown in Fig. 3, the variation of critical Rayleigh number with λ is quite similar for the rectangle and the ellipse.

7.2. *Applications to soil*

Table 2 gives values of α, ν, and κ for dry (air-filled) and saturated (water-filled) soil at 293 K. The values given for κ are typical values based on the extensive studies of de Vries [17, 18]. Using these values and taking $g = 9.8 \text{ m s}^{-2}$, we find $R = 11 \times 10^9\, k\theta A$ for dry soils and $R = 2.5 \times 10^9\, k\theta A$ for saturated soils. Units of k, θ and A are, respectively, m^2, Km^{-1}, and m^2. The permeability k varies in the range 10^{-11} m^2 or more for sands to 10^{-15} m^2 or less for fine-textured soils [19]. Evidently there is a wide combination of practical values of k, θ, and A which yield values of R satisfying the criteria for applicability of the present analysis.

The practical interest in these convective motions at small Rayleigh number centres on the flows themselves, and on the consequent passive convection of certain entities with small molecular diffusivity. In the case of water-filled soils, the transport in this way of nutrients, O_2, and CO_2 dissolved in the soil–water are of concern to plant scientists. The molecular diffusivity of these entities in water is of order 10^{-9} m^2 s^{-1}. Transport to plant roots plays an important role in plant nutrition, and the transport of O_2 and CO_2 to and from all subterranean plant parts is essential to maintenance of the respiration of such parts.

7.3. *Extensions of this study*

In conclusion, we note three directions in which extensions of this study may be made. Firstly, the approach may be carried over from 2-dim. flows to 3-dim. flows with cylindrical symmetry about a vertical axis. Secondly, the possibility offers of supplementing the present results by regular expansions in R with the 'coefficient' of each term an exact solution. Thirdly, our rotational invariance theorem has its analogue for free convection of Newtonian fluids. It is planned to treat these matters in later communications.

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LA CONVECTION LIBRE AUX PETITS NOMBRES DE RAYLEIGH DANS LES CAVITES POREUSES A SECTION TRANSVERSALE RECTANGULAIRE, ELLIPTIQUE, TRIANGULAIRE, ET D'AUTRE FORME

Résumé—On développe des solutions précises pour la convection libre aux petits nombres de Rayleigh dans les cavités poreuses à deux dimensions de forme différente provoquée par un gradient de température constant perpendiculaire au champ de gravitation. On établit le théoreme que de tels écoulements sont indépendants de l'orientation de la cavité. La longueur caractéristique (à rotation invariable) est donc $A^{1/2}$ (A—surface de la section transversale). On présente en détail des solutions précises pour les cavités rectangulaires et elliptiques à λ (rapport de l'axe long à l'axe court) arbitraire, ainsi que pour les cavités ayant la forme d'un triangle équilatéral. Le nombre de Rayleigh qui est critique á l'égard de l'application de l'analyse est de 1 environ pour les cavités à axe long horizontal et de λ^2 environ pour celles à axe long vertical. Il existe également de simples solutions précises pour cavités d'autres formes.

DIE FREI KONVEKTION BEI KLEINER RAYLEIGH-ZAHL IN POROSEN HOHLRAUMEN RECHTECKIGEN, ELLIPTISCHEN, DREIECKIGEN, SOWIE ANDEREN QUERSCHNITTES

Zusammenfassung—Für die durch ein einheitliches, zum Gravitationsfeld senkrechtes Temperaturgefälle in zweidimensionalen porösen Hohlräumen verschiedener Form bedingte freie Konvektion bei kleiner Rayleigh-Zahl werden Genaue Lösungen hergleleitet. Das Theorem, dass solche Strömungen unabhängig von der Hohlraumsorientierung sind, wird bewiesen. Die entsprechende (drehungsinvariante) Kennlänge ist demzufolge $A^{1/2}$ (A: Querschnittsfläche). Genaue Lösungen werden für rechteckige bzw. elliptische Hohlräume bei beliebigem λ (Verhältnis der langen zur kurzen Achse) sowie für Hohlräume, die die Form eines gleichseitigen Dreicks aufweisen, ausführlich beschrieben. Die für die Anwendbarkeit der Analyse kritische Rayleigh-Zahl beträgt bei Hohlräumen mit horizontaler langer Achse ungefähr 1 und bei solchen mit vertikaler langer Achse etwa λ^2. Es bestehen auch genau Lösungen fur Hohlräume anderer Formen.

СВОБОДНАЯ КОНВЕКЦИЯ ПРИ НЕБОЛЬШИХ ЧИСЛАХ РЕЛЕЯ ВНУТРИ ПОРИСТЫХ ПОЛОСТЕЙ С ПРЯМОУГОЛЬНЫМИ, ЭЛЛИПТИЧЕСКИМИ, ТРЕУГОЛЬНЫМИ, И ДРУГИМИ ПОПЕРЕЧНЫМИ СЕЧЕНИЯМИ

Аннотация — Точные решения разрабатываются для свободной конвекции при небольших числах Релея внутри пористых полостей различных форм обусловена равномерным температурным градиентом перпендикулярным к гравитационному полю. Теоремой установлено, что такие потоки независимы от ориентации полости. Соответствующая характерная (вращательно инвариантная) длина — $A^{1/2}$ (A — площадь поперечного сечения). Точные решения представляются подробно для прямоугольных и эллиптических полостей при произвольном λ (соотношение длинной оси к короткой) и для равносторонных треугольных полостей. Критическое число Релея для применения анализа для полостей с горизонтальной длинной осью — около 1 и с вертикальной длинной осью — около λ^2. Простые точные решения существуют также для полостей с другими форм.